

A Validated Approach to Teaching Engineering Mathematics

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***Abstract:** Teaching mathematics to engineers is a worldwide issue: this is clearly seen in the extent of relevant published work on the subject. There are many reasons for this, but a clear concern is how to actually approach the problem when developing new engineering mathematics modules for first year engineering students. This paper describes a systematic approach to this problem which was supported where possible by the best current pedagogical practices and based on previous experience gained by the authors in this area. This pedagogy is discussed and cited in detail and the efficacy of the endeavour presented along with data relating to the attendance, engagement, enjoyment and attainment of several different cohorts of students over a period of four years. The rationale behind this work was based on the need to radically improve learning in a specific first year engineering mathematics module. The new module was prepared and implemented in a student-centred teaching environment, conforming to a validated approach that the authors had developed and evaluated over three years (2005-2008). To motivate and engage the students, a key consideration was to ensure that the new mathematics module could integrate with the rest of the programme and espouse the same active and collaborative learning strategies inherent in the other more design orientated modules. It is concluded that there is no need to reinvent the wheel when it comes to effectively teaching mathematics to engineering students – adopting a thorough systematic approach to implement proven, existing, relevant pedagogy can suffice.*

***Keywords:** Engineering mathematics, declining mathematical skills, active and interactive learning, content, teaching, learning, assessment, Helping Engineers Learn Mathematics (HELM), student motivation, engagement and attainment.*

Introduction

The School of Mechanical and Aerospace Engineering at Queen's University Belfast (QUB) is striving to improve its student learning experience. A curriculum change plan was already being developed when the School became a collaborator in the CDIO Initiative (Crawley et al., 2007) in 2003. This is an innovative educational framework that provides students with an education stressing engineering fundamentals set in the context of Conceiving, Designing, Implementing and Operating (hence CDIO) real-world systems and products.

Context and Background

The School is now well underway in a process of reforming its engineering degree programmes in Mechanical, Manufacturing and Aerospace Engineering based on the CDIO principle and methodology. However, in 2004 the School introduced a new Product Design and Development (PDD) degree programme which was designed entirely on this CDIO ethos.

Extensive experience was gained in researching, developing and implementing the mathematics provision for this new PDD programme as the entry requirements were not as stringent as the School's other engineering programmes with regard to mathematical skills; an A-Level mathematics qualification was not required to enrol on the PDD programme.

The research, planning and preparation of the specific first-year engineering mathematics module within the School's PDD programme were described by McCartan and Hermon (2008). They clarify,

through many citations, that teaching mathematics to engineers is a worldwide issue mainly founded on the wide range of abilities and motivations of students currently enrolling in tertiary education. They discussed and cited their systematic and structured approach to developing this integrated engineering mathematics module, which was based on the best current pedagogical practices, innovations, resources and the guiding paradigm of active and interactive learning. The key points of their development plan to design and implement a successful first-year engineering mathematics module included: appreciating the benefits of integrating the learning of mathematics within the curriculum; fully understanding the potential student learning problems; researching and applying the best known practices in teaching tertiary mathematics; and utilising available, validated resources. These conformed well to the expert opinion in the UK that advocated all aspects of such effective support for mathematics in engineering degree programmes (Williamson et al., 2003).

The execution and evaluation of this aforementioned PDD engineering mathematics module, and specifically the assessment strategies employed, were also described by McCartan (2008), who reviewed assessment methods and tested the hypothesis that there can be a substantial impact on student learning by encouraging sufficient, relevant and productive out-of-class active learning activities. He established that continuous active learning can successfully improve student engagement and verified this with significant improvements in attainment.

All of this was achieved within the School over a three year period from 2005 to 2008 by carefully evaluating the PDD mathematics provision and continually developing it to improve efficacy, attainment, student satisfaction and ultimately retention. It should be noted at this stage that the typical size of the PDD cohorts during this period ranged from about twenty to forty students and all of the engineering mathematics teaching was provided within the School - so engineers were teaching engineers. It also meant that the evaluation of different learning and assessment strategies were feasible given the staff resources available.

Rationale and Challenge

The next logical step in the School's Curriculum reform plan, based on these successes in the PDD programme, was to review the existing first year mathematics provision on its other programmes. There was only one other first year mathematics module and this was common to the Mechanical, Manufacturing and Aerospace programmes. This was a long-standing engineering mathematics module which, in recent years, was showing poor scores for attainment and student satisfaction.

Therefore the challenge was to improve the learning and teaching in this specific first year engineering mathematics module where the students were not achieving the required skills and intended outcomes.

On average, over the last five years, there have been circa 130 students taking this module. The review of student attainment showed that nearly 40% of students who were sitting the exam in this module were failing it. On average, about 10% were not even taking the exam. The average mark for the exam was barely above the pass mark of 40%. Ironically, this was the first mathematics module for the Mechanical, Manufacturing and Aerospace programmes, with the majority of the learning outcomes being simply a revision of A-Level mathematics syllabi.

Further investigation revealed that very traditional didactic methods were being employed in this module. As a result, attendance was poor, student motivation was very low and this was clearly manifested in the assessment results and feedback. However, there was clear scope, based on previous experiences, to change both the teaching methods and assessment strategy employed and therefore improve the learning environment for the students and thereby improve attainment and possibly retention. Again, it should be noted that teaching on this module was to be provided within the School which provides the opportunity for a more integrated learning environment.

Objectives

This paper discusses the systematic pedagogical development of this first year engineering mathematics module based on the previous experiences with the smaller PDD classes. It defines the key areas of interest and describes the approach taken.

Essentially this was a pedagogical development project that involved the implementation of relevant pedagogical research in order to improve student attainment in a specific engineering mathematics module. The key objectives for this project were:

- To provide sufficient practice in the mathematical methods presented.
- To promote a deeper learning environment.

- To emphasise the relevance of mathematics to the degree programmes.
- To potentially develop other non-disciplinary skills such as professional, personal and interpersonal skills (according to the CDIO Syllabus).

The following sections discuss the pedagogy implemented in this module and present the efficacy of the endeavour along with data relating to the students' motivation, engagement and attainment in the course. In addition, the practical issues relating to delivering such an engineering mathematics module are discussed.

Process for Developing a New Engineering Maths Module

The process for planning, developing and implementing the new engineering mathematics module was based on experience gained from the PDD mathematics module and can be set out as follows:

- Obtain data on student background, abilities and skills.
 - Diagnostic testing
 - Learning styles inventories
- Establish context and relevance of module within the programme(s).
 - Choose Learning outcomes and content
 - Identify available resources
 - Ensure module integrates with other modules on the programme
- Determine learning and teaching methods.
 - Promote Active and interactive learning
 - Develop varied assessment strategy

The objectives of this process were essentially to develop teaching, learning and assessment practices that were student-centred. To do this it was necessary to be fully aware of the background and abilities of the students in order to tackle the ensuing pedagogical issues associated with such a curriculum design challenge. This was accomplished in two ways: mathematical diagnostic testing at entry as recommended by the Engineering Council UK (2000) and learning styles inventories (Hermon, 2007). The former provided information on the new students' mathematical skills and the latter indicated particular predominant learning preferences. Such information then helped provide clarity with regard to developing module content, teaching methods and effective assessment criteria that affords students a more balanced learning environment.

Relevant learning outcomes, skills and attributes for the new engineering mathematics module were identified by applying the CDIO methodology to course design. Armstrong and Niewhoner (2008) described this generic CDIO approach which is founded on teaching engineering in context and applying specific CDIO Standards (Crowley et al., 2007, p35) which emphasise not only the design of an integrated curriculum (Standard 3), but also the inclusion of integrated learning experiences (Standard 7).

The potential benefits of developing a mathematics module in such an integrated and streamlined learning environment were advocated years earlier by Carpenter and Schröder (1999) as being higher success and retention rates, higher quality graduates and shorter times to graduation. Their particular objectives for mathematics support in an integrated engineering curriculum were the introduction of key mathematical concepts "in context", and the elimination of unnecessary duplication throughout the curriculum.

Therefore, in a CDIO teaching environment the first key consideration was to ensure that the new mathematics module could integrate with the rest of the programmes and espouse the same learning strategies inherent in the other more design orientated modules, which was considered essential if the students were to stay motivated and engaged throughout.

The specific content of the new engineering mathematics module was finalised by conducting interviews with all the appropriate teaching staff on the programmes, which also follows best practice with regard to integrating the learning of mathematics within an engineering curriculum (Wilcox and Bounova, 2004).

The teaching methods were varied to facilitate active and interactive learning in class, thus conforming to CDIO Standard 8 (Crowley et al., 2007, p35). In addition, an effective assessment strategy was implemented to promote and encourage out-of-class active learning, based on previous experience (McCartan, 2008).

Outline of the New Mathematics Module

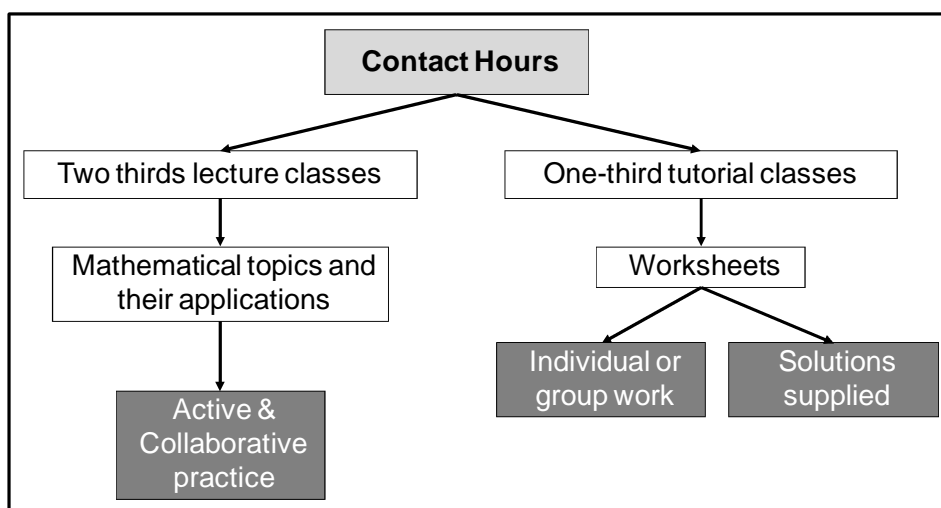


Figure 1: Overview of New Mathematics Module

The new module was similar to the original in as far as its structure would comprise two-thirds lecture classes and one-third tutorial classes (Figure 1). However, the lecture classes were designed to be more active and interactive with the students being tasked to complete problems during the class, individually and in groups, to enhance their learning. This facilitated instant feedback on a regular basis to both the students and the lecturer. A set of comprehensive printed notes were supplied to the class with a significant number of blank areas to complete the tasks. It should be pointed out that flexible learning spaces are an advantage over traditional lecture theatres when applying this approach.

More support was provided for the tutorial sessions which enabled classes of smaller numbers to be formed. Again, the students were encouraged to work in groups to potentially maximise their learning.

The learning outcomes for the module are shown in Table 1 along with more detailed information on the topics covered.

Table 1: Module Learning Outcomes

Learning Outcomes	Topics
<u>Knowledge and understanding of:</u>	Indices, Polynomial Expressions, Scientific notation, Exponentiation and logarithms.
Equation manipulation	Simplification and factorisation, Solving linear Equations, Solving quadratic equations,
Polynomials	Solving polynomial equations, Partial fractions.
Trigonometry	Trigonometry: Inverse trig functions, Solving right angle triangles, Trig identities, Sine rule, Cosine rule.
Functions (and how they represent engineering phenomena)	Complex Numbers: Algebra of complex numbers, Solution of polynomial equations with complex roots, Argand Diagrams, Polar form of complex numbers, Exponential form of complex numbers, Series expansion of trigonometric and exponential functions, De Moivre's theorem.
Complex numbers	Differentiation: Gradients, Differentiation from first principles, Table of derivatives, Evaluating derivatives, Higher derivatives, Differentiating products and quotients, Chain rule, Parametric differentiation, Implicit differentiation.
Non-linear equations	Applications of Differentiation: Maxima and minima.
Differentiation	Integration: Table of integrals, Rules of integration, Definite integrals, Area bounded by curves, Integration by parts, Integration by substitution and using partial fractions.
Integration	Application of Integration: Centres of mass, Moments of inertia.

Learning & Teaching Resources

The relevance of mathematics to engineering was continually highlighted throughout the course and supported were necessary by pertinent worked examples.

The main text for the module (Croft and Davidson, 2004) was chosen for three reasons: firstly, it espoused the interactive approach which was desired; secondly, it was perfectly integrated and supported by two excellent resources, namely, the Helping Engineers Learn Mathematics curriculum development project (HELM, 2010) and the on-line mathematics support centre, MathCentre (2010); and thirdly, the authors already had successful experiences of supporting mathematics learning using all of these resources (McCartan and Hermon, 2008; McCartan, 2008; McCartan and Hermon, 2009). In particular, it is worth noting that the HELM project has been very well disseminated by the HELM team and others (HELM, 2010), detailing many examples of how it is being applied.

To further support learning, the HELM resources were installed on the School's Virtual Learning Environment (VLE) and also supplied to the students on CDs. Throughout the module, continual links and pointers were given to the related topics in both the HELM and MathCentre resources to reinforce this learning support.

Assessment Strategy

The assessment strategy is a key factor in any learning environment. Gibbs and Simpson (2003) and Rust (2002) both advocate this by focusing on carefully directing student learning inside and outside the classroom, and designing assessment that encourages learning. Obviously this all has to be achieved within the context of "constructive alignment" as advocated by Biggs (1999).

In the School of Mechanical & Aerospace Engineering at QUB, this paradigm has been successfully used to develop successful assessment strategies with the smaller PDD classes of twenty to forty students. The challenge and objective of this specific curriculum development project was to implement it in a new engineering mathematics module for around one hundred and thirty students. Significantly more resources in terms of personnel were required: three members of teaching staff and six postgraduate helpers in total – compared to one member of teaching staff and two postgraduate helpers for the equivalent PDD module.

The assessment strategy implemented in the new module involved focusing on out-of-class learning via tutorial worksheets and regular mini-class-tests (circa twenty minutes duration) based directly on the tutorial questions and the worked examples and tasks from the active and interactive lectures. This coursework comprised of four class-tests and was allocated forty percent of the marks for the module with the rest apportioned to a traditional end of semester examination. It should be noted that the HELM resources can be used to facilitate Computer Assisted Assessment (CAA) (Davis et al., 2005), which the author's have broached (McCartan and Hermon, 2009), but this is a work in progress (see Further Work section at end).

New Module Efficacy

Did this new engineering mathematics module meet its objectives? Were the students, engaged, motivated and did they attain the intended learning outcomes? This section endeavours to answer these questions by providing a detailed evaluation of the new module in the form of both summative and formative data:

- Assessment Results
- Student Feedback

Assessment Results

The new engineering mathematics module was first delivered in 2009 and then again in 2010. The assessment statistics for this new module are shown in Figure 2 and compared to data from the previous two years of the old module – i.e. 2007 and 2008 (the statistics before 2007 were very similar and are omitted for clarity). The bar chart in Figure 2 shows four succinct bits of information on the horizontal axis: The total number of students enrolled for the module each year; the number of students that failed the module each year; the number of students that were absent from the examination each year; and the average mark for the module each year. In 2009 and 2010 the recorded failures were only 6% and 8% of the respective cohorts compared with nearly 40% in the previous two years. In 2009 and 2010 the absences from the examination reduced by over 50% compared to the previous two years. In 2009 and 2010 the average mark for the module rose by almost 50% from 43 to 63 and 61 respectively. It should be noted that, as part of the official evaluation process for this new module, the class-tests and final examinations in 2009 and 2010 were checked by a specially formed mathematics committee to ensure parity with the previous years.

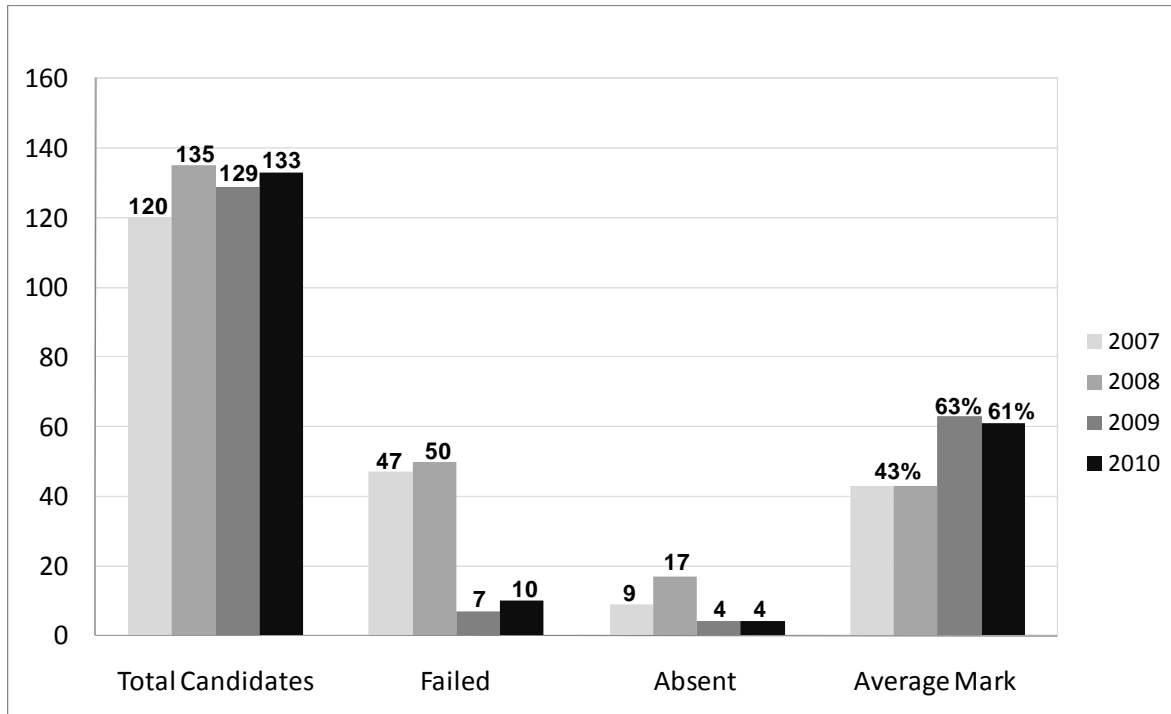


Figure 2: Assessment Information over a Period of Four Years

Student Feedback

In line with the School’s official module evaluation process the students were asked to fill in a pro-forma questionnaire at the end of the new first-year engineering mathematics module. There are two sections on the questionnaire, the first asking for a score in relation to particular statements regarding the module (Table 2), to gauge overall satisfaction and identify areas of concern, and the second requiring the students to provide written comments to two open questions.

Table 2: Module Evaluation Questionnaire & Scoring

Scoring	
5: Strongly Agree	4: Agree
3: Neutral or N/A	2: Disagree
1: Strongly Disagree	
Questions	
1	The module aims and objectives were stated clearly
2	The module's relevance to my degree programme was explained clearly
3	The module was well structured
4	The examination procedures & assessment criteria were stated clearly
5	The module content was presented clearly
6	The teaching was effective and relevant to the module aims and objectives
7	The pace of the lectures was appropriate to my needs
8	Sufficient examples and applications were provided in lectures
9	There was good support at tutorial classes
10	There was good support at practical classes
11	There was sufficient consultation available for all elements of the module
12	There was good interaction and feedback between students and lecturer
13	There was clear guidance on coursework objectives and arrangements
14	There was adequate time for completion of coursework tasks
15	The lecturer provided me with helpful and timely feedback on my work
16	The workload was appropriate to the module size

Table 2 shows the sixteen statements that the students were asked to rate according to their own personal opinions using the scoring system provided (also shown in Table 2). Their scores were collated, averaged and factored (by 20) to a percentage value indicating how well they agreed with each statement. These results are shown in Figure 3, which is a bar chart representing the percentage agreement scores for each of the sixteen questions. It is clearly evident from this simple analysis,

based on the first part of the questionnaire, that the students were satisfied with the module contents, the teaching methods, the assessment methods, the feedback and the lecturer's contributions to their learning. The results indicated a satisfaction level of over 70% for all aspects of the module (the dashed line in Figure 3).

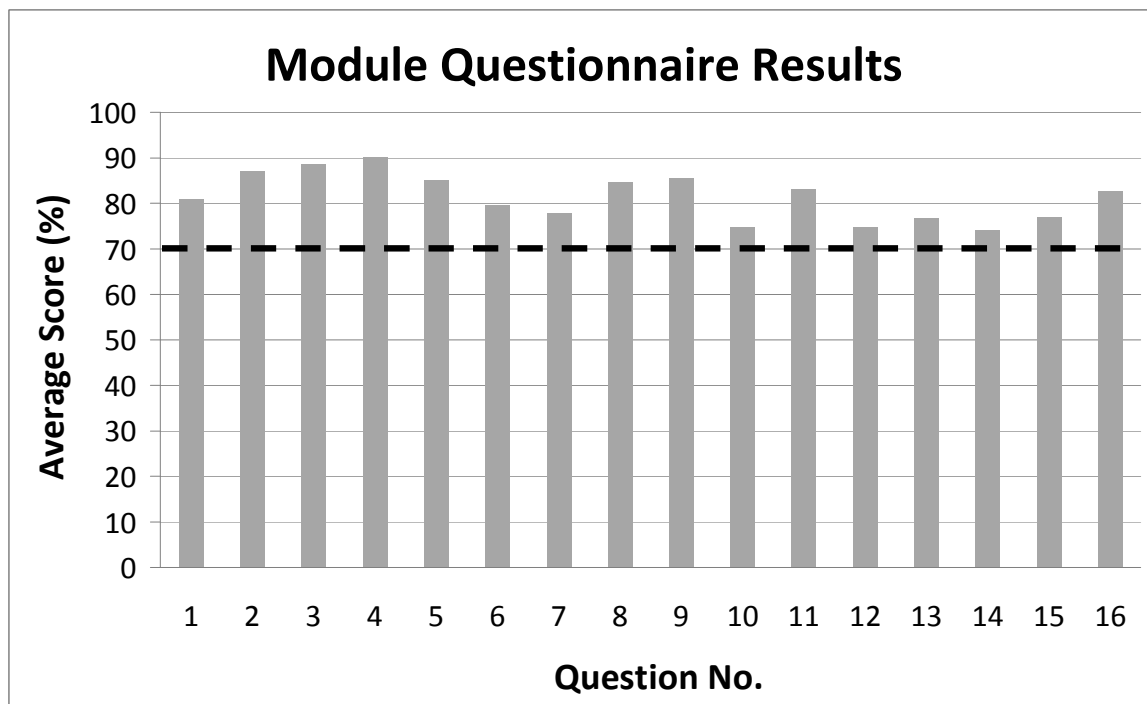


Figure 3: Results from Module Questionnaire Sheets

The second part of the questionnaire indicated that the students actually appreciated and even enjoyed the active and interactive teaching and learning methods employed. The students conveyed this message by responding to two open questions:

- Please indicate the most satisfying aspect(s) of this module
- Please indicate the least satisfying aspect(s) of this module

The students' responses also targeted several other areas of the module that particularly pleased them:

- Worked examples and tasks within the lectures.
- Lecturing style, pace, hand-out notes.
- Class tests and continual feedback.
- Balance between exam and coursework.
- Tutorial support.

They also flagged up a couple of areas for improvement:

- More worked examples
- Easy access to tutorial solutions

These comments provided further evidence on the efficacy, engagement and attainment of the students, thus indicating what was working well in the new module and what required revision.

Conclusions

A carefully designed method, based on previous experience, to plan, prepare and implement a new engineering mathematics module has been successfully applied.

An active and interactive teaching approach, combined with a continuous assessment scheme to encourage student learning has been shown to improve attainment in this module. Furthermore, the formative feedback from the students was very positive in relation to all the teaching and learning methods employed.

Employing the best practice in relation to the pedagogy appropriate for teaching mathematics to engineering students involved fully supporting the students by:

- Diagnostic testing at outset.
- Using Learning styles inventories.
- Implementing an active and interactive approach to learning and teaching.
- Continually highlighting the relevance of mathematics to engineering.
- Integrating the mathematics module with the other engineering modules.
- Exploiting the relevant available texts and online resources.
- Promoting learning through the assessment strategy.

The active and interactive learning approach, combined with the continuous assessment strategy, provided instant individual and collective feedback to the students and the staff. In addition, it offered an enjoyable and constructive learning environment which fostered a more positive attitude towards learning mathematics.

However, there were some potential issues with this approach:

- The course preparation required significantly more work by the staff.
- The continual assessment regime employed required more work by the staff.
- In-class active and collaborative activities required a bigger commitment from the staff.

Further Work

Work is already in progress to develop and implement CAA and Computer Aided Learning (CAL) within the School (McCartan and Hermon, 2009). Relevant pedagogical research in this area recommends such a tactic and links perfectly with the approach described above. The following four citations corroborate this claim.

Croft and Ward (2001) have explored (CAL) to motivate and encourage students by providing instant feedback on their progress. They have also implemented CAA to add force and momentum to an existing assessment strategy which promotes continual learning outside the class.

In the same vein, Croft et al. (2001) report on a specific implementation of CAA in a first year engineering mathematics module that has been thoroughly tested, evaluated and proven. They show that CAA works very well as a tool to promote learning when it is associated with coursework credit. The feedback from their students is very positive.

Two years ago Janilionis and Valantinas (2008) presented a detailed account of active learning methods being employed to teach engineering mathematics at Kaunas University of Technology in Lithuania. They adopted the same interactive approach to lectures as discussed in this paper, and emphasised the importance of virtual learning environments (VLEs), CAA and software applications to produce a more attractive learning experience.

Further corroboration is presented by Challis and Gretton (2008) who advocate the use of computers and graphical calculators to provide multiple representations of mathematical concepts quickly, correctly and easily. In their opinion, developing such a rich, accessible set of mathematical tasks for the students challenges them to make decisions and interpret, explain and reflect on possible solutions. As a final point they conclude that such tasks should be enjoyable.

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