P101

A Quantitative Approach to Identifying Threshold Concepts in Engineering Education

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Abstract: *Research by Meyer and Land (2003) introduced the notion of 'Threshold Concepts' – concepts that students initially find troublesome, but which, when understood, transform and integrate ways of thinking about their respective discipline. The identification of threshold concepts in engineering may help guide the design of core courses and the approaches used in teaching mastery components. However, the identification of such concepts has proved rather troublesome in itself, with ongoing debate as to value of the construct in real teaching contexts. This study uses frequent student survey to look for the characteristics which may be indicative of threshold concepts; attention is given to some key first year undergraduate-level engineering concepts. A question-set is developed, and electronic handheld voting devices used to regularly quantify the student self-perception of understanding, which is further compared with a test of actual understanding. In addition, a retrospective survey and interview are used together with chronological record of students' self-perceptions and test results to identify transforming instants. Finally, a favourable set of survey questions is proposed for identifying the defining characteristics of potential threshold concepts.*

Introduction

Meyer and Land introduced the classification 'threshold concept' (Meyer and Land, 2003) to describe concepts that act as gateways, or 'portals' of understanding. Such concepts are characterized by five properties:-

- Initially troublesome to students
- Transformative of understanding
- Integrative – linking with other concepts in previously unseen ways
- Irreversible – cannot be unlearned easily
- Bounded – defining the boundary of an area of understanding

Mayer and Land suggest that once a student has been introduced to such a concept, they enter a state of '*liminality*' (Meyer and Land, 2003) – a state associated with being 'stuck' and not possessing a mastery of the concept – until the necessary transformation of understanding has taken place and the 'threshold' is crossed. Being in a state of liminality can be unsettling, and some students may not make the changes in understanding required to progress from that state. Once the threshold is crossed, however, new areas of knowledge and understanding become accessible.

Progress in crossing the threshold may be associated with emotional response – an enlightenment or 'aha' moment when the penny drops and the new way of thinking takes over. Certainly the state of liminality has been associated with emotional arousal and discomfort (Kiley and Wisker, 2009).

Different levels of understanding have been described by Perkins (Perkins, 2008) as *possessive* – being able to recall items of information, *performative* – understanding is expressed as thought and action in the context in which it was learned, and *proactive* – knowledge can be applied to different contexts, requiring understanding at a more abstract level. There are parallels between transitions from one level to another and the transformation associated with threshold concepts.

It has been proposed that concepts that possess these powerful characteristics should therefore be explicit focal points in the design of curricula and in pedagogic activity (Australian National University).

There has been a good deal of research to determine which concepts in the varying disciplines have the characteristics of threshold concepts. Much of this is based on student feedback by interview and questionnaire, by teachers' experience of student reaction to concepts introduced during taught courses, and their own recollections of understanding such concepts.

Concepts that have been considered as having threshold concept properties include complex numbers in maths and engineering (Meyer and Land, 2003), object-orientation and recursion in computer science (Zander et al., 2008; Rountree and Rountree, 2009) and opportunity cost in economics (Meyer and Land, 2003). However, methods of identifying threshold concepts are still vague and ad-hoc, to the extent that the validity of the threshold concept model is still debated.

This research uses quantitative feedback from student questionnaire and survey as well as qualitative feedback in an attempt to develop a systematic approach to identifying concepts in computer science and basic engineering education having threshold properties. Work done by Davies (Davies, 2003) identified that the design of questions used to explore student understanding of concepts should be informed by what scholars believe should be taught, to reduce the influence of the variation in student experience. The questions developed for the questionnaire are, however, non-specific in terms of material, addressing only student experience. Whilst this does of course lead to issues of subjectivity, the rationale behind taking a stochastic approach with a reasonably sized population was that it can reveal trends that are not necessarily caught by low sample size student interview.

Three investigations were carried out :-

1. An compulsory online questionnaire was given to Imperial College Department of Computing year 1 students at the end of a class - the aim of this investigation was to test the ability of a question set to distinguish between concepts that had been identified as having threshold properties and those that had been identified as not having those properties (See appendix 1 for list of questions)
2. A tracking of perception of understanding of Imperial College Bioengineering year 1 students on a vibrations module. The aim of this second investigation was to look for longitudinal changes in understanding that might to identify states of liminality or instants of transformation.
3. A questionnaire was given to the same Department of Bioengineering students. The aim of this third investigation was to test the discriminating power developed in the first investigation on a different set of concepts.

Methods

1. Department of Computing First Year Questionnaire

All 102 students in the 2008-9 first year cohort of the Department of Computing were required to complete a survey at the end of a term in which they had been introduced to some of the basic tenets of computer programming.

Five concepts were chosen for investigation

1. Recursion
2. Binary trees
3. Memory Allocation
4. Object Orientation
5. Prolog Backtracking

Two of these (1 and 4) had been previously identified as having the characteristics of threshold concepts (Zander et al., 2008; Rountree and Rountree, 2009), and the other three were considered by academics in the Department teaching the course not to have these characteristics, but to be concepts that could be understood with a linear application of time and effort.

Fifteen questions were designed for the survey, based upon the first four primary defining characteristics of threshold concepts:-

a. Troublesome nature
b. Irreversibility of understanding
c. Integration of other concepts
d. Transformation of understanding

The questionnaire comprised explicit questions on threshold concept properties and levels of understanding. See appendix 1 for a list of the questions.

Questions 2,8 and 11 addressed transformation, questions 4 and 14 addressed troublesome nature, questions 5,6,9,12 and 13 addressed integration, questions 10 and 15 addressed irreversibility and questions 1,3 and 7 examined levels of understanding.

The students were required to submit their responses to the fifteen questions for each of the five concepts. The responses were made, via a webpage, on a scale with 11 possible selections. The scale had three labels – one end of the scale was labelled 'strongly agree', the other end was labelled 'strongly disagree' and the central value of the scale was labelled 'Neutral'.

Three of the characteristics of threshold concepts - irreversibility of understanding, integration of other concepts and transformation of understanding – can only be tested when the threshold has been crossed, or at least when students are emerging from a liminal state. Therefore, a question which was an indicator of understanding (Q7 – "I still do not understand the concept") was used to screen the results. Students whose responses ranged from 'strongly agree' to 'neutral' that they still did not understand the concepts had their responses to the other questions excluded from the data. After the screening, the sample sizes were as follows:-

1. Recursion – 77 respondents
2. Binary trees -74 respondents
3. Memory Allocation – 65 respondents
4. Object Orientation – 68 respondents
5. Prolog Backtracking – 57 respondents

At the end of the questionnaire, the students were asked to complete free-form text boxes in answer to the following two questions:-

1. In all the programming courses what was/were the hardest thing(s) to understand?
2. Which concept or concepts do you feel were most important to your understanding of programming?

The responses to the questionnaire were statistically analysed to indentify which of the questions were the most significant predictors of whether or not a concept lay in the group of concepts proposed as threshold concepts. Two different methods of statistical analysis were compared –stepwise linear discriminant analysis based on the minimization of Wilks' Lambda and binary logistic regression. Whereas both methods can identify significant discriminators, binary logistic regression makes no assumption that the data is normally distributed.

2. Tracking of Perception of Understanding on a Vibrations Module

The survey described above was carried out after the teaching had been delivered. Further insight was desired into the longitudinal progression of understanding as concepts were introduced and learning activities took place. Monitoring this progress might expose states of liminality and moments of transformation. To this aim, regular surveys were taken of a student cohort on a module in free and forced vibrations in the first year of a course in Biomedical Engineering in the Department of Bioengineering.

The surveys were taken using 'clickers' – this enabled surveys to be taken quickly without taking too much lecture time. The students could respond to questions by selecting one of six possible options.

Each student was allocated a particular clicker with a unique radio-frequency ID, so that individual progress could be tracked as well as the overall progress of the cohort. No adjustment to the teaching of the course was made in response to the results of the survey so that the research did not affect the learning process relative to previous years - this may have influenced results. This does, however, leave scope for comparison in future years. The average sample size of student respondents per survey was 45.

At six different points during the five weeks of the module, students were asked to give a response that they thought most closely described their level of understanding of each of the following six concepts:-

1. Differential equations – general solution
2. Differential equations – boundary conditions
3. Complex numbers
4. Phasors
5. Free vibrations
6. Forced vibrations

Their response was chosen from the following options:-

1. I fully understand – let's move on
2. I get it, but could use some practice
3. I mostly get it, but go over it just once more
4. I'm starting to get something – keep explaining
5. I'm really not very sure about this
6. What on earth are you talking about?

The responses had to be made within 15 seconds - this meant that the surveys could be taken without a significant time overhead at the start of a lecture.

Learning activities on the module comprised two lectures, one study group and a laboratory each week. The six sample points were not equally distributed in terms of time or contact hours. The distribution was as follows:-

Table 1. Intervals between sample points

Interval between samples	Time	Lectures	Study groups	Labs
1-2	1 working day, 1 weekend	Complex numbers(1)	Complex numbers(1)	None
2-3	8 working days, 1 weekend	Vectors(1) Phasors(1) Complex impedance(1)	Vectors(1)	Oscilloscopes(2)
3-4	1 working day, 1 weekend	Free vibration(1)	Free vibration(1)	None
4-5	5 working days	Free vibration(1)	Free vibration(1)	Free vibration(1)
5-6	5 working days, 1 weekend	Free vibration(1) Forced vibration(2)	Forced vibration(1)	Free/forced vibration(1)

The results of the tracking data were used to identify students who had indicated either sudden jumps in their perception of understanding of specific concepts – possibly indicating transforming instants, or students whose perception of understanding after the material had been covered had remained low, possibly indicating a continuing state of liminality. These students were asked specific questions about their experiences whilst tackling the concepts.

At the end of the module, the students were given an oral examination on their understanding of free and forced vibrations, and their test scores were compared against their perception of understanding.

3. Vibrations Module Questionnaire

At the end of the vibrations module, a questionnaire was given to the same students whose progress had been tracked. This questionnaire was similar to that given to the Department of Computing students. A subset of eight of the original fifteen questions was used. The 11-point scale had two endpoint labels – ‘strongly agree’ and ‘strongly disagree’, but no ‘neutral’ label. The eight questions were asked of each of the six concepts tracked. The sample size of respondents was 46.

Results

1. Department of Computing First Year Questionnaire

Three of the questions were identified by the linear analysis as being significant discriminators:-

1. I would find <the concept> easy to explain to another student
2. Understanding <the concept> has transformed the way I think about the subject
10. I would find <the concept> hard to forget now I understand it

The first question had a negative co-efficient – possibly indicating that proposed threshold concepts were harder to explain than non-threshold concepts. The second two questions had positive coefficients – proposed threshold concepts were transforming and hard to forget. These three questions correctly classified 65.2% of responses into the two groups of proposed threshold and non-threshold concepts. The results were again analysed by binary logistic regression using only the three primary discriminators with a 67.4% success rate:-

Table 2. Classification of responses into proposed categories by logistic regression (TC=threshold concept)

		Predicted		
	Observed (proposed)	TC	Non-TC	Percentage Correct
Step 1	Threshold Concept	80	65	55.2
	Non-threshold concept	46	149	76.4
	Overall Percentage			67.4

2. Tracking of Perception of Understanding on a Vibrations Module

Figures 1-6 below show the statistics of the cohort responses taken at the six sample points described in table 1. The sample points were taken before lectures for logistical reasons. A higher value of response indicates a lower perception of understanding according to the scale previously described. The arrows indicate the point at which the subjects were formally introduced during the module.

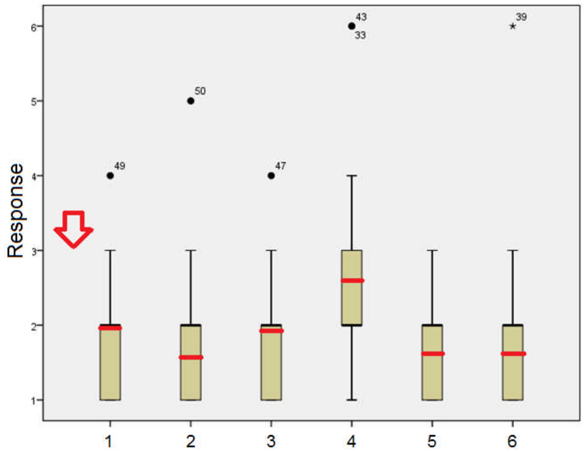

Figure 1. Differential Equations General Solution

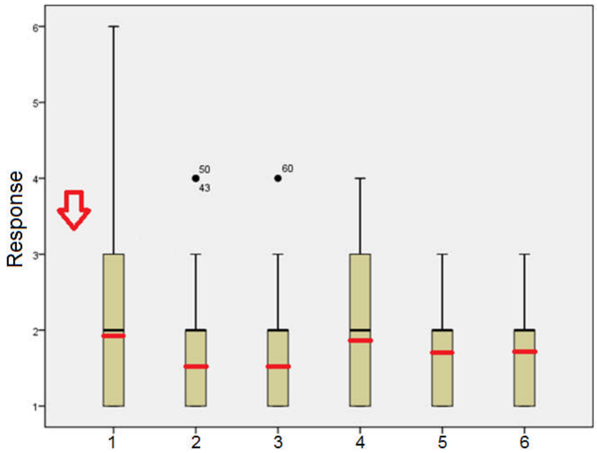

Figure 2.Differential Equations Boundary Conditions

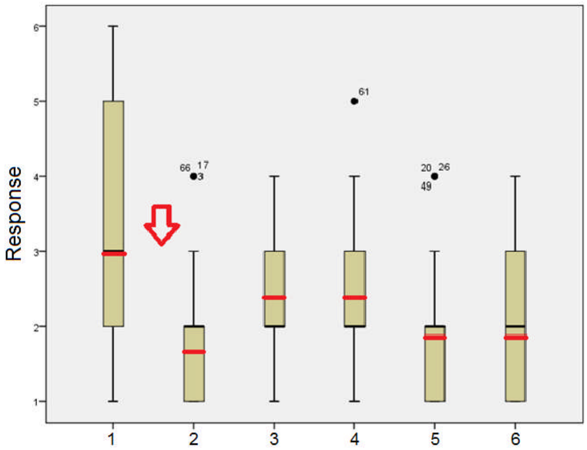

Figure 3. Complex Numbers

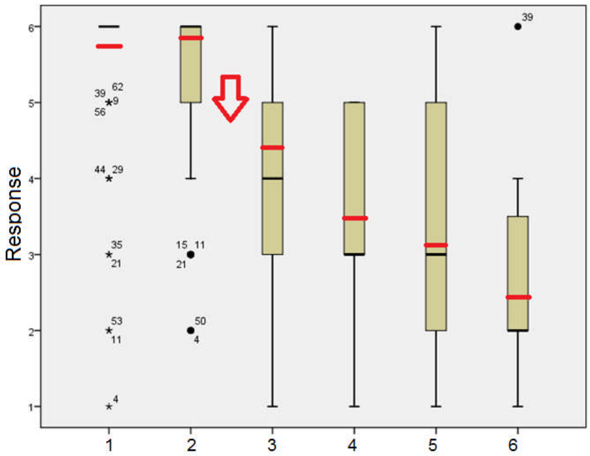

Figure 4. Phasors

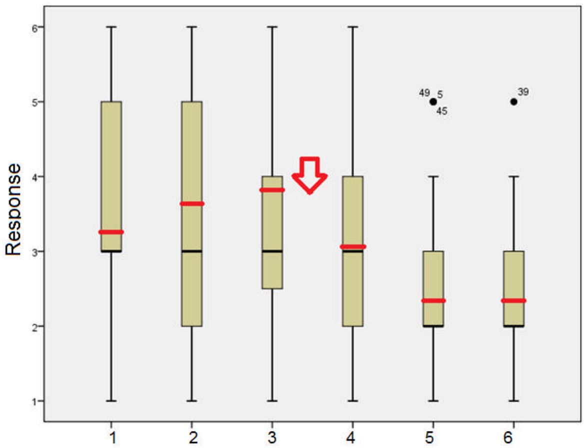

Fig 5. Free Vibration

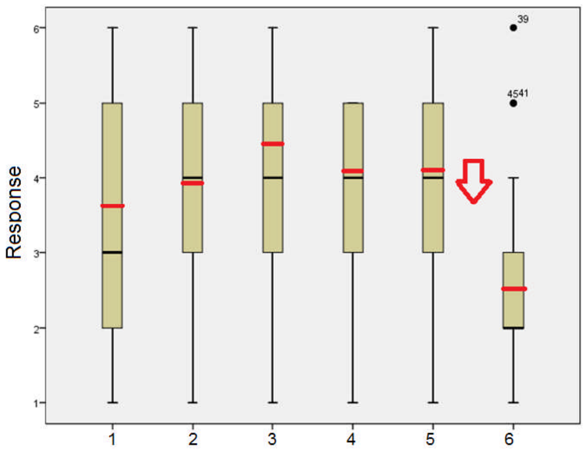

Fig 6. Forced Vibration

The students took an oral examination on their understanding free and forced vibrations. The table below compares the students' perception of understanding in free and force vibration to the oral exam score. The perception score ranges from 1 for highest perception to 6 for lowest and the exam marks were in percentage.

Table 3. Correlation of exam mark with perception of understanding in free and forced vibration recorded in survey 6 (N=36 responses for free vibration, N=37 responses for forced vibration)

		Exam score	Free vibration perception	Forced vibration perception
Exam score	Pearson Correlation	1	-.382*	-.416*
	Sig. (2-tailed)		.021	.010
	N	40	36	37
Free vibration perception	Pearson Correlation	-.382*	1	.741**
	Sig. (2-tailed)	.021		.000
	N	36	37	34
Forced vibration perception	Pearson Correlation	-.416*	.741**	1
	Sig. (2-tailed)	.010	.000	
	N	37	34	38

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

3. Vibrations Module Questionnaire

The logistic regression discrimination process developed in the first investigation was applied to the responses to the six concepts on the vibrations course with the following results:-

Table 4. Classification of responses for each vibrations module concept

	Threshold concept	Non-threshold concept
Differential Equations - General Solution	10%	90%
Differential Equations – Boundary conditions	17%	83%
Complex Numbers	26%	74%
Phasors	52%	48%
Free Oscillation	29%	71%
Forced Oscillation	44%	56%

Discussion

1. Department of Computing First Year Questionnaire

Linear discriminant analysis and logistic binary regression demonstrated that three of the questions in the questionnaire distinguished between the group of proposed threshold concepts and the group of proposed non-threshold concepts with a success rate (67.4%) similar to the entire set of questions (68.5%). These three questions included one designed to test irreversibility (10), one designed to test transformation (2) and one designed to test levels of understanding (1).

The 11 point scale used for response had three labels – 'strongly agree' and 'strongly disagree' at the endpoints and 'neutral' at the midpoint, therefore care is required when drawing conclusions by treating it as parametric. Not all of the distributions of responses were normal, and as logistic regression does not make the same assumptions about the normality of the distribution, it would be a more appropriate method of analysis, but the close agreement between the logistic regression success (67.4%) and the discriminant analysis success (65.2%) was nevertheless encouraging.

The free-form comments endorsed the consideration of object-orientation and recursion as troublesome concepts. The third and fourth most common replies to the question 'In all the programming courses what was/were the hardest thing(s) to understand?', were recursion (14% of responses) and object-orientation (14% of responses). The first two most common replies were Prolog in general (25% of responses) and the Haskell programming language (16% of responses).

Replies to this question included:-

> *'The harder concepts (e.g. recursion) when they were fresh and new to me'*
>
> *'The object oriented way of thinking.'*
>
> *'recursion was a little difficult to understand. I couldn't apply the 'LEAP OF FAITH' very confidently.'*

In reply to the question 'Which concept or concepts do you feel were most important to your understanding of programming?', the most common responses were object-oriented programming (43% of responses) and recursion (38% of responses).

> *'object oriented programming without a doubt;*
>
> *'Main topics such as recursion, objects etc. Things that as a concept are not programming language specific.'*
>
> *'recursion!!'*

Questions that remain to be answered include:-

Why questions testing the integrative property were not significant.

Why the question (1) on the difficulty in explaining the concepts to another student was significant – this may be an expression of the troublesome nature of threshold concepts.

Were the students applying different questions to different sub-concepts within the main concept? For example, would a student find complex number arithmetic easy to explain to another student, but still not find the exponential form of complex numbers transformative?

2. Tracking of Perception of Understanding on a Vibrations Module

Differential equations, complex numbers and free and forced vibrations had been introduced to some students at school. The box-plots show an initial broad range of perception of understanding that narrows as the subject is taught, as is hoped.

Free and forced vibrations would not have been covered previously in the mathematical depth that they were covered on the vibrations module. The average perception of understanding is initially high but decreases – an indication that the subject is being dealt with in more depth than at school, as students realize that they don't understand the concept at the new level – before then increasing again.

The peak on the perception scale at survey number 4 for differential equations – general solution, was identified as the point that second-order differential equations were introduced – only first-order equations had been previously covered.

The concept of phasors was new to almost all students, and a steady increase can be seen in perception of understanding with time as phasors are initially taught, and then used as tools to understand real-life situations in the lab. The range of perception of understanding, however, remains wide until they are used as the central mathematical description of forced vibrations, taught between surveys 5 and 6.

One of the main aims of the tracking investigation was to identify instants of transformation and ongoing liminal states.

One student's perception of understanding of phasors over the six sample points had been 6-4-4-4-4-4 –indicating a possible liminal state. The student's reply to the question 'What were the hardest things [in the module] to understand?' was:-

> *"One difficult thing to understand was the concept of phasors. Initially it caused some trouble for me to visualise the way phasors work, but when we reached the topic of forced oscillations I understood how useful phasors are. Although I was familiar with the basic idea forced oscillations from my studies at A-Level Physics, some of the principles were again difficult to understand, but by understanding phasors I could see how easily the solutions were worked out."*

From this student's statement, we may have expected a rise in perception of understanding for at least the last sample point. This student's reply, however, does indicate a troublesome nature, a liminal period and an opening of new terrain when understood. Phasors are certainly integrative – relying on connections between complex numbers, sinusoidal properties and exponentials. The application of complex numbers to real components – inductors and capacitors - is initially alien knowledge to students (Meyer and Land, 2003). For this student, the concept had to be visualized – this may point to the use of animations – possibly applets – as a primary aid in the teaching of phasors.

Another student, whose perception of understanding of phasors had progressed as 6-6-5-5-3-2, which may have indicated a transforming instant between surveys 4 and 5, replied to the question 'Was there a specific moment when you suddenly 'got' phasors, or was it gradual?' as follows:-

> *"It was gradual but i never really 'got it' since to me 'got it' is defined as mastered it. Im never comfortable with a theory unless i feel perfect in it and able to solve any reasonable question."*

This reply does not point to a transforming instant. As a tool for tracking overall progress of a cohort and identifying points at which there is synergy between concepts – for example between phasors, complex numbers and forced vibrations, the clicker surveys can make valuable contributions to module planning. For tracking of individual student progress, there may be some subjectivity in either the students' clicker responses, or in their replies to the individual questions, which warrants further investigation.

The correlation between test score and perception of understanding demonstrates that it is possible to use the students' perception as a measure of actual understanding.

3. Vibrations Module Questionnaire

Application of the discriminating tool developed in the first investigation to the concepts of the third investigation classified 'phasors' in the category of concepts having threshold concept properties, and the other five as concepts as non-threshold concepts.

The finding that phasors are classified as threshold concepts correlates with the fact that virtually none of the students had come across them before – this was not the case with the other concepts. This may have overlapping effects on the outcome of the responses – that they had no peers to ask for help, that the predominant student feeling about them was uncertainty, and that they didn't feel embarrassed to rate the concept as troublesome as they were not in a minority.

However, the statistical results and the comments of the student in the previous section point to the possibility that phasors may be threshold concepts.

Due to the findings of previous research, it was expected that complex numbers would be classified as threshold concepts, but this was not the case. To explore this further, one student whose tracking of perception of understanding of complex numbers had progressed as 5-2-2-1-2-2 was asked the question 'Had you come across complex numbers previously? They tend to be quite disturbing when you first meet them - was this the case for you, and if so, what helped you come to terms with them?'

> *"I did not come across them in A-level. I did not find them difficult. I worked example in the past papers and on blackboard and textbooks like stroud."*

Students' prior understanding of complex numbers may have also biased their response to the questionnaire – their memory of how troublesome or reversible the understanding may have lapsed. However, when applying the discrimination process to students whose tracking data indicated that they had not come across them before, the split in responses between the two categories was found to be the same as the entire population (74% non-threshold concept, 26% threshold concept).

This work shows some underlying attributes of TCs in certain troublesome topics, but also indicates the difficulty of validating the broad definitions of TCs in a classroom context.

Future Work

Further analysis of the data captured in these three investigations may help to answer some of the questions raised, and prompt a refinement of the question set. To minimize the effect on results of student prior understanding, the techniques described in this paper will be applied to concepts that students meet in their second year of engineering, which will be new to the entire cohort.

The significance of the questions on difficulty of explanation, irreversibility and transformation in the analysis indicate that these properties may be quantified to some degree by direct questionnaire. However, the questions on the integrative property of threshold concepts were not significant in the discrimination, which may indicate that this more sophisticated property cannot be expressed as response to a questionnaire. Work by Kinchin and Hay (2000) on concept mapping has been identified by Mayer and Land (2008) as a possible means of exposing the understanding of interconnections between concepts, and the evolution of those connections. The use of concept maps as tools of learning as well as tools of evaluation and assessment requires further research. Further comparison with test results will be made to compare perception of understanding with actual understanding

Clickers have been identified as a valuable tool for efficiently quantifying student understanding. It is likely that they can be used to expose further trends and parameters in the student learning process. Work is proceeding to identify further areas of use.

Appendix 1 - Questionnaire

Questions used for the survey of Department of Computing students:-

1. I would find <the concept> easy to explain to another student
2. Understanding <the concept> has transformed the way I think about the subject
3. I could apply <the concept> to new areas
4. <the concept> wasn't any more difficult to understand than any other concept
5. <the concept> is something that I now see as central to the subject but which was rather tricky to understand
6. Understanding <the concept> caused me to better see some relationships between topics that I could not see before
7. I still don't really understand <the concept>
8. Understanding <the concept> felt quite emotional like 'aha now I see what that means'
9. Lots of things came together to understand <the concept>
10. I would find <the concept> hard to forget now I understand it
11. understanding <the concept> made me feel more like a computer scientist
12. It was difficult to see how <the concept> fits in with any other subject
13. Once I understood <the concept> other previously studied subjects made more sense
14. Understanding <the concept> was troublesome
15. I will have to keep refreshing my understanding of <the concept>

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