The Application of Computer Algebra Software in the Teaching of Engineering Mathematics

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Abstract

The overall aim of the Mini-Project was to make use of computer algebra software (Maple is available campus-wide across Loughborough University) in the teaching of maths to a mixed-ability class of first-year Chemical Engineers. The intention was to engage both strong and weak students in an interactive and stimulating study of maths, without requiring them to slog through turgid algebra to obtain every result. A series of eight interactive workbooks were developed to cover the majority of the first-year maths syllabus in Chemical Engineering. The software is not simply a black-box symbolic manipulator. Whilst it does contain simple commands to perform complex operations, one of its strengths is that it also allows line-by-line analysis, similar to a pen and paper solution, but without the fear of making algebraic errors. For example, rule-based methods for algebraic, differential and numerical operations may be applied and line-by-line solutions of ODEs may be conducted. This approach allows students to concentrate on the underlying mathematical principles, for example in the solution of first and second order ODEs, without worrying about detailed manipulations. It also allows the student to explore different solution methods for a given problem.

Background

Students entering engineering UG programmes increasingly come from diverse mathematical backgrounds; some have a good understanding of the methods, but find implementation difficult, whereas others struggle with simple algebraic manipulations. Without proficiency in maths, these students will be unable to cope with quantitative analyses of engineering problems. To increase access and broaden participation in engineering, whilst maintaining retention, we must find means of engaging students with the required maths, making it fun to learn and providing tools they can use in more advanced applications. An aim of the project has been to widen participation in engineering programmes and particularly to help students from disadvantaged backgrounds.

In the Chemical Engineering Department at Loughborough University, our approach to the teaching of first-year maths has been initially to use some lecture time to revise A-level material, but to concentrate on giving plenty of practice on abstract and applied problems, using (i) pen and paper and (ii) CAS software. The latter is innovative in that it provides the able students with a new skill they can apply in other contexts; for the weaker students it takes the drudgery out of algebraic solutions, allowing them to concentrate more fully on the underlying principles of the maths being taught. Once familiar with the use of the software, the students may progress to more advanced maths, again concentrating on understanding the basic methods without obstruction from detailed manipulations. Knowledge gained from this early exposure to CAS software will usefully be applied in the remainder of the course.

Methodology

The Mini-Project developed an integrated series of Maple® worksheets for use in our first-year mathematics modules, to assist with the revision of A-level materials (algebra, differentiation and integration) and with the learning of new topics such as first- and second-order ODEs, series, limits and partial differentiation. The sheets were written based on Maths lecture notes prepared by the author and various people were employed to assist with developing the worksheets:

- a BEng Chemical Engineering student worked on the first two sheets as his final year R&D project
- an MEng Chemical Engineering student was employed for 10 weeks over a summer vacation
- a Project Officer from the Engineering Education Centre worked for about 3 weeks
- a graduate Maths student worked for about 4 weeks (as part of an Engineering Education Centre project).

The first worksheets were written to familiarise the students with the application of Maple to the solution of simple algebraic problems, using the software capabilities to perform specified operations on equations. Students are shown how to use the short-cut commands available in Maple for a number of basic maths operations, but the emphasis has been on developing the appropriate steps required in a solution. This encourages the students to explore different methods to obtain an answer. Thus the Maple sessions have added an extra dimension to more conventional examples classes in maths and other subjects, where students are still expected to solve problems by hand.

Each worksheet section comprises an introductory discussion explaining the methods to be practised during that session, followed by a number of well-chosen and graded examples. Through the wording of the questions, the students are encouraged to pursue a line-by-line approach to the solution of a maths problem. Initial worked examples are used to illustrate this approach (these are made interactive using Maple commands and can be illustrated graphically). At the end of each section, there are examples to be solved which require the students to generate the appropriate input statements. Some of the initial examples are abstract, and so are generally applicable to all engineering disciplines. The later examples are specific to chemical engineering and show how a physical description of a problem may be formulated into a set of equations, which may subsequently be solved. These type of questions build on other first-year topics from the Chemical Engineering course, but could easily be changed to suit a variety of engineering disciplines. All of the problems have a "Student workspace", followed by hyperlinks to worked solutions, provided in .pdf format and stored on the secure VLE at Loughborough. Students can only access these solutions from the worksheet hyperlinks and the use of .pdf format means that it not possible to cut and paste the Maple commands back into the worksheet.

Many problems in engineering have to be solved using numerical methods. Maple® provides an almost seamless interface between analytical and numerical solutions and so some worksheets provide practice in techniques such as numerical integration, data regression and solution of linear and non-linear equations.

We have used Worksheets 1 to 4 in Semester One of the first-year Maths module in Chemical Engineering, running one Maple session per week. The largest computer lab we have available has about 40 PC seats, which is too small for our whole class. Therefore three-hour sessions were booked each week; the first two hours of the session were supervised by a member of academic staff and a post-graduate teaching assistant. Although the students are only required to attend for one of these hours, many stay on for longer and continue working on the Maple sheets in their own time. We allow students to work at their own pace (although we warn those that are not making fast enough progress). Depending on the student, worksheets 1 to 4 will provide sufficient material for about 10-12 one-hour classes. Although we have not run worksheets 5-8 with our first-year class, they should also provide about 10-12 one-hour classes, so that the whole set is sufficient for a one-year module. The worksheets are delivered via Loughborough University's secure VLE and students are able to download the worksheets and access the solutions from any computer with an internet connection. Loughborough University has a campus-wide Maple license; students can obtain a CD containing the Maple software from the Chemical Engineering department and are allowed to load the software onto their home computers for their personal use. Almost all of our first-years students now have access to Maple on their personal computers.

The students are positive and enthusiastic about learning maths through the use of Maple software, but obviously they look for some reward in terms of marks towards the module. The Maple worksheets count towards the coursework element of the module. Each worksheet is marked on a pass/fail criterion, but students are allowed to revise their submissions until a pass is achieved. In Semester Two, a course work exercise is set which requires the students to solve a series of more complex problems using the Maple software (see web site); these are marked and returned to the students along with suitable feedback.

Issues

- Familiarity with the software is initially a barrier to student success in solving the first-year maths problems. However, our experience has been that the students pick up the general principles through explanations in the text of the worksheets and by seeing examples. Moreover, they rapidly learn how to use the 'Help' pages - in the first few sheets, hyperlinks are provided to direct the students to relevant help pages, but later they are expected to search for themselves to determine the syntax and arguments for commands and functions.
- Very few of our students have experience of writing software input commands in any language or for any application. Thus a further initial difficulty is in paying attention to detail, so that the required syntax is precisely followed. This does cause some initial difficulties and frustration, but the demonstrators in the lab classes now have sufficient experience to be able to spot errors and to suggest corrections. However it is a useful learning experience for students, that attention to spelling and correct use of brackets, etc. are important in computer applications.
- In our first-year classes we have a few dyslexic students who found it difficult to scroll back through the worksheets and then scroll forward again to apply the methods to a problem solution. We overcame this by providing these students with printed versions of the worksheets (with all commands executed), so that they could have the relevant section open in front of them whilst tackling a new problem. Maple has a zoom facility that enlarges the text for students with visual impairments.
- This type of exercise is only ultimately useful if the methods learnt in the first-year, for solving maths problems using Maple, are built upon and further developed in subsequent parts of the course. Such development requires the co-operation of colleagues, who need to become familiar with the maths/Maple methods if they are to incorporate such ideas within their own teaching. This remains a challenge in our department and the ongoing aspect of this project is to persuade colleagues that they can tackle more interesting and more complex problems, without fear of mathematics, using the skills that are already available within our student cohort. Modules which have a strong mathematical component can benefit from taking such an approach: for example Fluid Mechanics, Heat and Mass Transfer, Process Control and Reaction Engineering are areas where in the future we expect to see our students using Maple solutions in their coursework and projects.

Benefits

The first year that we ran these classes there were no worked solutions to the problems, so the students often got stuck and could progress no further, even though they were willing to work for longer in their own time. Now, with the solutions available through hyperlinks, even when working in their own time, the students can get help and as a results we saw a marked improvement in the rate at which sheets were completed. Competence in solving unseen problems, with no solutions available, is tested using a coursework assignment later in the academic year.

The overarching benefit is that these worksheets have provided an alternative means to engage student in studying mathematics and have given them an extra set of tools and skills for performing symbolic or numeric calculations in other aspects of their engineering degree.

Evidence of Success

The student feedback through the Maple classes and via their feedback forms has been thoroughly positive. The strongest students in the class are generally enthusiastic and competent in mathematics - they find Maple to be a new challenge, which is enjoyable and stimulating to learn. In doing so, they are also revising and remembering many of the topics from their A-level courses and they are getting further practice at tackling abstract and applied problems. The weaker students at maths also seemed to pick up the ideas about the software fairly quickly, although they struggled more with the mathematical concepts. The supervised classes are useful for this category of student, since they can seek individual attention and generally the teachers can sort out maths misconceptions as well as Maple difficulties (often they are linked). It is clear that these students are enthused by using a different tool to do their maths and as a result they seem prepared to work harder on their own to complete the sheets. Success in using Maple, requires a certain level of

mathematical and logical understanding and so the very weakest students are probably not helped much by using these worksheets; they almost see them as another barrier put in their way. Sometimes their demotivation towards maths is so deep-seated that they can generate very little interest in learning new computer-based methods.

In the last two years, there have been almost no failures in Chemical Engineering in the mathematics modules. Thus we would claim that the use of these worksheets has been one means by which we have improved our retention rates and the success rates at passing mathematics at the first-attempt.

How Can Other Academics Reproduce This?

The basic material in Worksheets 1 to 8 cover much of any first-year engineering maths module and the examples used would be relevant to a number of disciplines. The worksheets are modular in construction and make use of Maple's structure of sections, sub-sections, etc. It would be straightforward to add / remove sections and tailor them to suit specific requirements. Similarly it would be easy to remove any unsuitable problems and add new ones which were closer to the engineering context of that discipline. Overall the worksheets provide a framework for teaching maths through computer algebra software that could be extended to more advanced stages.

Reflections

- Some improvements could be made to the overall structure of the workbooks. After having used them for two years, I would prefer to break them into a greater number of small sheets, ideally so that each one could be completed in a week. I think this would add to the sense of student achievement, whilst making it easier to monitor their progress throughout the Semester.
- The first versions of these sheets probably made too much use of detailed line-by-line methods of analysis - for example, in performing algebraic manipulations. In their second or third versions, some of this has been removed and the methods now rely on more direct commands for Maple to solve algebraic equations. The calculus sheets (Worksheets 3 and 4) continue to make use of Maple's StudentCalculus package, which is intended for teaching purposes and makes use of the application of a series of rules (*e.g.* chain rule, product rule, etc. for differentiation). This is useful, but perhaps on reflection the students do not appreciate the true power of the software to carry out these types of operations, without being concerned over the details.

References

For distribution to students at Loughborough University, the worksheets and solutions can be accessed through the secure VLE (the Learn Server) at:
https://learn.lboro.ac.uk/cg/04cga011/content/contents/Tutorials.htm

For the purposes of open dissemination to other interested academics a mirror of these pages has been posted at:
http://www-staff.lboro.ac.uk/~cgcdr/LTSN

This web site also provides access to the following information:

- the slides from a talk given by the author at the 'Engineering Mathematics Examples and Case Studies Swap-Shop', Thursday 29 January 2004, LTSN Engineering, Loughborough University: "*The application of computer algebra software in the teaching of engineering mathematics*"
- coursework problems to be solved using Maple: these would be complex, but not impossible to solve by hand; using Maple, much of the mathematical complexity is reduced.